Left-right symmetry and heavy particle quantum effects.

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Abstract

We have renormalized a classical left-right model with a bidoublet, and left and right triplets in the Higgs sector. We focus on oblique corrections and show the interplay between the top quark, heavy neutrinos and Higgses contribution to the muon Δr parameter. In the SM, custodial symmetry prevents large oblique corrections to appear. Although in LR models there is no such symmetry to make vanish the quadratically diverging terms, we have shown, that heavy Higgses contributions to Δr are under control. Also the top contribution to Δr, quite different from that in the SM, is discussed. However, heavy neutrinos seem to give the most important contributions. From oblique corrections, they can be as large as the SM top one. Moreover, vertex and box diagrams give additional non-decoupling effects and only concrete numerical estimates are able to answer whether the model is still self-consistent.

1 Introduction

Many non-standard models have already been considered in the literature at the quantum level. They inevitably involved new physical parameters. For instance, if we extend the Higgs sector by an additional Higgs doublet, then mass splitting between neutral and charged scalars can be examined [1]. In the MSSM, supersymmetric particles must also be taken into account and the analysis is much more sophisticated [2]. This has nothing to do with the gauge sector (the gauge group is the same as in the SM), but with the amount of particles in the game. Additional problems appear in models where heavy neutrinos are introduced. This has been examined in the frame of the SM with additional isosinglet neutrino fields (which build up Dirac neutrinos) [3]. Then, the interplay between heavy neutrinos and light standard particles on the one hand, and nondecoupling effects [4] on the other, is important . Finally,

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non-standard models with extended gauge groups have been considered. Let us mention only the papers by *Senjanovic and Sokorac* [5], where the left-right symmetric $SU(2)_L \times SU(2)_R \times U(1)$ has been work out, with conclusion that scalar particle effects do not decouple in low-energy processes. This has been done, however, in a model without heavy neutrinos [1] and without renormalization (a class of diagrams has been chosen, which, after summing, yields a finite answer). At this point, we should also mention a paper by *Soni et al.* [8], where useful limits on the additional charged gauge boson mass and mixing have been obtained. Note, that their analysis does not require a renormalization procedure (finite box diagrams). A renormalization scheme has been proposed in [9]. It has, however, a different nature from the present, and the consequent analysis bears no similarity.

In this paper we consider a left-right symmetric model where all of the effects mentioned above, come simultaneously into play. No numerical estimates will be given. Here we focus only on the renormalization procedure in a simple, practical renormalization framework based on ideas from the SM. It can be subsequently used both in low- and high-energy physics. To make our presentation clear we discuss and give exact relations for oblique corrections. The rest, i.e. influence of heavy neutrinos and Higgses on vertex and box diagrams will be shortly commented, and a detailed analysis will be postponed to [10]. As a laboratory we use the muon decay process. Let us remind, that the precision experiments, such as the muon decay, put extremely stringent constraints on the oblique corrections [11]. These latter, due to the custodial symmetry, depend only weakly on the Higgs boson mass in the SM [12]. However, there has always been the danger that the higher order corrections will take the LR model down. Now, in our case, there is no symmetry to make vanish the quadratically diverging terms. As the tree-level phenomenological bounds put the Higgs boson masses in the TeV range, there is a real risk that it will be impossible to accommodate all of the data, since the radiative corrections will grow indefinitely large. The present work shows that the situation is "reasonable" and without concrete fits the model cannot be ruled out.

The organization of the paper is the following. In the next section the most important ingredients of the model are described (details can be found in [13–16]). Next, the renormalization framework will be given and in a subsequent section, a quantitative discussion will be presented. We end up with conclusions.

[1] In the light of new Superkamiokande data [6], it is still attractive to allow the see-saw mechanism to operate [7].

2 The model

There exists a large class of LR models. They differ by the symmetries imposed and by the details of the Higgs sector. Much has been written about it in the literature [14–17], we will therefore justify our choices only briefly.

The basic characteristics of the model in question follow from the symmetries. The first is given by the gauge group, which is:

$$SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}. \tag{1}$$

It defines the gauge field content, and after the choice of matter fields has been made, the interaction of these fields with the gauge bosons and between themselves (to the extent, that further symmetries may still constrain).

It is by now agreed [14,15] that a minimal model structure should contain these scalar fields:

(1) two Higgs triplets $\Delta_{L,R}$, with quantum numbers $(1,0,2)$ and $(0,1,2)$ respectively, to generate Majorana neutrino masses through the see-saw mechanism,
(2) a Higgs bidoublet Φ, with quantum numbers $(1/2, 1/2^*, 0)$, to generate charged fermion masses.

We shall adopt the following convenient representation:

$$\Delta_{L,R} = \begin{pmatrix} \delta^+_{L,R}/\sqrt{2} & \delta^{++}_{L,R} \\ \delta^0_{L,R} & -\delta^+_{L,R}/\sqrt{2} \end{pmatrix}, \quad \Phi = \begin{pmatrix} \phi^0_1 & \phi^+_1 \\ \phi^-_2 & \phi^0_2 \end{pmatrix}. \tag{2}$$

Even after the choice of the fields, the allowed lagrangians will still have much freedom left. Additional constraints follow from the left-right symmetry:

$$W_L \leftrightarrow W_R, \quad \Psi_L \leftrightarrow \Psi_R, \quad \Delta_L \leftrightarrow \Delta_R, \quad \Phi \leftrightarrow \Phi^\dagger, \tag{3}$$

where $W_{L,R}$ are gauge fields associated to the left and right $SU(2)$ gauge groups, and $\Psi_{L,R}$ are left and right fermion fields. Imposing this symmetry leads not only to several simplifications, but also to a restoration of parity invariance at high energies (in the unbroken phase). This should be considered the most important argument for its introduction.

The most general Higgs potential allowed by the adopted symmetries, that we consider in this paper, was discussed in [14–16].

The scalar fields can develop vacuum expectation values through the spontaneous symmetry breaking mechanism:

$$< \Delta_{L,R} >= \begin{pmatrix} 0 & 0 \\ v_{L,R}/\sqrt{2} & 0 \end{pmatrix}, \quad < \Phi >= \begin{pmatrix} \kappa_1/\sqrt{2} & 0 \\ 0 & \kappa_2/\sqrt{2} \end{pmatrix}. \tag{4}$$

A very careful analysis of the symmetry breaking pattern has been given by *Deshpande et al.* [15]. Surprisingly, it turns out that due to see-saw type relations between the vacuum expectation values and the coupling constants, the possible values of the parameters are strongly constrained, to the point that avoiding fine tunings requires setting several of them to zero. Therefore, as it has been argued, the most convenient results can be obtained with $v_L = 0$ and β terms in the Higgs potential put to zero (see [15]).

Several other approximations are in order. Since we do not consider CP violation effects, we assume all of the Higgs potential parameters to be real, which yields real VEVs. Furthermore, strong suppression of $FCNC$, requires that either of $\kappa_{1,2}$ be very small, or vanishing [15,18]. To simplify the expressions, the one-loop analysis will be done with $\kappa_2 = 0$ [2]. The general character of the radiative corrections will not be changed by this last assumption.

There are twenty real fields at our disposal. Since some of them are charged, there should be fourteen distinct fields (unconnected by symmetries). This number is further reduced, as some of them will become Goldstone bosons for the gauge fields (their number is fixed by the symmetry breaking pattern, to be four). Thus, we end up with ten physical fields. Let us now write them in terms of the original scalars.

We begin with four neutral scalars H^0_a, $a = 0, 1, 2, 3$, and two neutral pseudoscalars $A^0_{1,2}$ [3]:

$$\phi^0_1 = \frac{1}{\sqrt{2}}(H^0_0 + i\phi^{0i}_1), \tag{5}$$

$$\phi^0_2 = \frac{1}{\sqrt{2}}(H^0_1 + iA^0_1), \tag{6}$$

$$\delta^0_R = \frac{1}{\sqrt{2}}(H^0_2 + i\delta^{0i}_R), \tag{7}$$

$$\delta^0_L = \frac{1}{\sqrt{2}}(H^0_3 + iA^0_2), \tag{8}$$

[2] due to the symmetry of the Higgs potential, the model is symmetric with respect to replacement $\kappa_1 \leftrightarrow \kappa_2$.

[3] the lack of mixing between the physical scalars is only approximate and follows from the large difference of scales between κ_1 and v_R

where δ_R^{0i} and ϕ_1^{0i}, are massless and thus functions of the Goldstone fields:

$$\begin{pmatrix} \delta_R^{0i} \\ \phi_1^{0i} \end{pmatrix} = \begin{pmatrix} -s_g^n & -c_g^n \\ c_g^n & -s_g^n \end{pmatrix} \begin{pmatrix} G_1^0 \\ G_2^0 \end{pmatrix}, \tag{9}$$

with (see next section for definitions of the gauge sector mixing parameters):

$$s_g^n = -\frac{gv_R}{M_{Z_1}}\frac{s}{c_M} = \frac{g\kappa}{2M_{Z_2}}(c_M c - s/c_W), \tag{10}$$

$$c_g^n = \frac{gv_R}{M_{Z_2}}\frac{c}{c_M} = \frac{g\kappa}{2M_{Z_1}}(c/c_W + c_M s). \tag{11}$$

The charged sector contains two singly charged Higgses $H_{1,2}^{\pm}$, two singly charged Goldstone bosons $G_{1,2}^{\pm}$, and two (physical) doubly charged Higgses $\delta_{L,R}^{\pm\pm}$:

$$\delta_L^{\pm} = H_1^{\pm}, \tag{12}$$

$$\delta_R^{\pm} = s_g^c H_2^{\pm} \pm i c_g^c G_2^{\pm}, \tag{13}$$

$$\phi_1^{\pm} = c_g^c H_2^{\pm} \mp i s_g^c G_2^{\pm}, \tag{14}$$

$$\phi_2^{\pm} = \mp i G_1^{\pm}, \tag{15}$$

with:

$$c_g^c = \frac{\sqrt{2}v_R}{\sqrt{\kappa_1^2 + 2v_R^2}}, \qquad s_g^c = \frac{\kappa_1}{\sqrt{\kappa_1^2 + 2v_R^2}}. \tag{16}$$

By orthogonality of the respective mixing matrices, s_g^n, s_g^c, and c_g^n, c_g^c, define sines and cosines of some mixing angles.

We will not specify details of the gauge-Higgs and Yukawa interactions. They can be found in [16]. The charged gauge-lepton current is:

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}}\left(\overline{N}\gamma^{\mu}K_L P_L l W_{1\mu}^{+} + \overline{N}\gamma^{\mu}K_R P_R l W_{2\mu}^{+}\right) + h.c. \quad . \tag{17}$$

Here K_L, K_R are neutrino mixing matrices. It is justified by present experimental data and neutrino mass generation mechanisms, that light neutrinos couple strongly to leptons through left currents, and heavy neutrinos through right currents. Even more than that, since in all considered processes light neutrino masses can be neglected, we may assume that the interaction between leptons and light neutrino states is diagonal. The most important consequence

is that one loop corrections to muon decay are inserted to the tree level W_1 diagrams (not to the W_2 diagrams, and $W_1 - W_2$ mixing is not considered).

The neutral current is [19]:

$$
\begin{aligned}
\mathcal{L}_{NC} = \frac{e}{2s_W c_W} & \Big(\bar{l}\gamma^\mu (A_L^{1l} P_L + A_R^{1l} P_R) l Z_{1\mu} + \overline{N}\gamma^\mu (A_L^{1\nu}\Omega_L P_L + A_R^{1\nu}\Omega_R P_R) N Z_{1\mu} \\
& + \bar{l}\gamma^\mu (A_L^{2l} P_L + A_R^{2l} P_R) l Z_{2\mu} + \overline{N}\gamma^\mu (A_L^{2\nu}\Omega_L P_L + A_R^{2\nu}\Omega_R P_R) N Z_{2\mu} \Big) \\
& - e \left(\bar{l}\gamma^\mu l A_\mu \right),
\end{aligned} \tag{18}
$$

where:

$$
\Omega_L = K_L K_L^\dagger, \quad \Omega_R = K_R K_R^\dagger, \tag{19}
$$

Here also, we can state that Ω_L couples diagonally light to light neutrino states, whereas Ω_R couples (but in general non-diagonally) mostly heavy states. Couplings between light and heavy states can be neglected in most but a few specific cases where the diagram is proportional to heavy neutrino mass squared and must be treated separately.

3 Renormalization

From the point of view of the SM, precision tests based on four-fermion reactions may be considered complete, if we forget about a few recompilations of results. The case is much different for the LR model. Since the analysis is simpler, when there is no gauge boson final states, we shall begin here a systematic work on radiative corrections in this specific situation. Thus, the only wave functions that need to be renormalized are fermionic (plus the photonic one, as we shall see in a moment). Let us remind, that the wave function renormalization constants serve only the purpose of properly normalizing the amplitudes. The real choice comes with the physical input parameters. Forget for a moment Higgs and Yukawa sectors and focus on the gauge one. The free parameters are:

$$
g, \quad g', \quad \kappa_1, \quad \kappa_2, \quad v_R. \tag{20}
$$

All physical parameters (mixing angles and masses) can be expressed in their terms, namely gauge boson mixing matrices defined as follows ($\kappa_+ = \sqrt{\kappa_1^2 + \kappa_2^2}$):

$$\begin{pmatrix} W_L^\pm \\ W_R^\pm \end{pmatrix} = \begin{pmatrix} \cos\zeta & \sin\zeta \\ -\sin\zeta & \cos\zeta \end{pmatrix} \begin{pmatrix} W_1^\pm \\ W_2^\pm \end{pmatrix}, \tag{21}$$

$$\begin{pmatrix} W_L^3 \\ W_R^3 \\ B \end{pmatrix} = \begin{pmatrix} c_W c & c_W s & s_W \\ -s_W s_M c - c_M s & -s_W s_M s + c_M c & c_W s_M \\ -s_W c_M c + s_M s & -s_W c_M s - s_M c & c_W c_M \end{pmatrix} \begin{pmatrix} Z_1 \\ Z_2 \\ A \end{pmatrix}, \tag{22}$$

where:

$$c_W \equiv \cos\theta_W, \quad s_W \equiv \sin\theta_W, \quad c_M \equiv \frac{\sqrt{\cos 2\theta_W}}{\cos\theta_W}, \quad s_M \equiv \tan\theta_W$$
$$s \equiv \sin\phi, \quad c \equiv \cos\phi. \tag{23}$$

gives

$$\tan 2\zeta = -\frac{2\kappa_1\kappa_2}{v_R^2}, \qquad \sin 2\phi = -\frac{g^2\kappa_+^2\sqrt{\cos 2\theta_W}}{2\cos^2\theta_W (M_{Z_2}^2 - M_{Z_1}^2)}. \tag{24}$$

The angle θ_W, which we call in analogy to SM, the Weinberg angle, is connected to the electric charge and the coupling constants g, g':

$$g = \frac{e}{\sin\theta_W}, \qquad g' = \frac{e}{\sqrt{\cos 2\theta_W}}. \tag{25}$$

Similarly, the masses of the gauge bosons are given by the following equations:

$$M_{W_{1,2}}^2 = \frac{g^2}{4}\left(\kappa_+^2 + v_R^2 \mp \sqrt{v_R^4 + 4\kappa_1^2\kappa_2^2}\right), \tag{26}$$

$$M_{Z_{1,2}}^2 = \frac{1}{4}\Big(\left((g^2\kappa_+^2 + 2v_R^2(g^2+g'^2)\right) \tag{27}$$
$$\mp \sqrt{(g^2\kappa_+^2 + 2v_R^2(g^2+g'^2))^2 - 4g^2(g^2+2g'^2)\kappa_+^2 v_R^2}\Big).$$

So, finally, we can use in the renormalization procedure the following set of physical parameters

$$e, \quad M_{W_1}, \quad M_{W_2}, \quad M_{Z_1}, \quad M_{Z_2}. \tag{28}$$

The electromagnetic coupling constant and the light boson masses are known and natural, just the same way as they are in the SM. Whether to use, in

numerical analyses, M_{W_1} or take it from the muon decay remains an issue reserved for future.

Let us recapitulate the main points of the scheme:

(1) we only renormalize the fermion wave-functions, no gauge boson renormalization constant is introduced, except for the photon (and this only to define the electric charge),
(2) the masses are renormalized on-shell (*i.e* the poles of the respective propagators are fixed at the physical masses),
(3) the mixing angles of the gauge boson sector are renormalized using their relation to the gauge boson masses.

We now have to make a remark about the self-consistency of our scheme when we assume that $\kappa_2 = 0$. There is then no mixing between the charged gauge bosons at tree-level (see Eq.(24)). A divergent contribution to this mixing shows up at one-loop level through fermion loops. Thus, although we can safely keep $\kappa_2 = 0$, a counter-term will still be necessary. However, in muon decay we do not have to bother about it, since tree level diagrams with W_2 transitions are negligible [20].

We now return to the Higgs sector. We assumed that certain coupling constants were zero, without any symmetries imposed. This would lead to non-renormalizability. If we were to consider the renormalization of this sector, additional counter-terms would have to be introduced. Fortunately, for the muon decay case this will not be necessary.

The scheme described above is an extension of the work of *Sirlin* [21].

3.1 Fermion Propagator Renormalization

Let us turn to the fermion propagator renormalization. If we introduce the notation $-i\Sigma_{ba}$ for the irreducible contribution to the transition from a to b, and decompose it according to the Lorentz structure:

$$\Sigma_{ba}(p) = \hat{p} P_L \Sigma^{\gamma L}_{ba}(p^2) + \hat{p} P_R \Sigma^{\gamma R}_{ba}(p^2) + P_L \Sigma^{1L}_{ba}(p^2) + P_R \Sigma^{1R}_{ba}(p^2), \quad (29)$$

then the following set of renormalization conditions can be given (the hat denotes renormalized quantities):

$$m^l_a \hat{\Sigma}^{\gamma L}_{ba}(m^{l\,2}_a) + \hat{\Sigma}^{1R}_{ba}(m^{l\,2}_a) = 0,$$

$$m^l_a \hat{\Sigma}^{\gamma R}_{ba}(m^{l\,2}_a) + \hat{\Sigma}^{1L}_{ba}(m^{l\,2}_a) = 0,$$

$$m^l_a \hat{\Sigma}^{\gamma L}_{ab}(m^{l\,2}_a) + \hat{\Sigma}^{1L}_{ab}(m^{l\,2}_a) = 0,$$

$$m^l_a \hat{\Sigma}^{\gamma R}_{ab}(m^{l\,2}_a) + \hat{\Sigma}^{1R}_{ab}(m^{l\,2}_a) = 0, \tag{30}$$

for $b \neq a$, and:

$$\hat{\Sigma}^{\gamma L}_{aa}(m^{l\,2}_a) = \hat{\Sigma}^{\gamma R}_{aa}(m^{l\,2}_a),$$

$$\hat{\Sigma}^{\gamma L}_{aa}(m^{l\,2}_a) + \hat{\Sigma}^{\gamma R}_{aa}(m^{l\,2}_a) + 2m^{l\,2}_a \left(\hat{\Sigma}^{\gamma L\,\prime}_{aa}(m^{l\,2}_a) + \hat{\Sigma}^{\gamma R\,\prime}_{aa}(m^{l\,2}_a)\right) \tag{31}$$

$$+2m^l_a \left(\hat{\Sigma}^{1L\,\prime}_{aa}(m^{l\,2}_a) + \hat{\Sigma}^{1R\,\prime}_{aa}(m^{l\,2}_a)\right) = 0,$$

for the diagonal case. These equations can be satisfied by introducing mass counter-terms and matrices of left(right) wave function constants. The situation can be simplified if the transition occurs through light particles. In the following, we will neglect the heavy neutrino contributions [4]. Thus, all the self-energies are diagonal (as already discussed, K_L neutrino mixing matrix connected with light neutrinos is assumed to be diagonal). We now need only diagonal wave function renormalization constants. They can be expressed through (see [1], for the definition of Δ):

$$\begin{aligned} \delta Z^{l_a}_L = {} & \Sigma^{\gamma L}_{aa}(m^{l\,2}_a) + m^{l\,2}_a \left(\Sigma^{\gamma L\,\prime}_{aa}(m^{l\,2}_a) + \Sigma^{\gamma R\,\prime}_{aa}(m^{l\,2}_a)\right) \\ & + m^l_a \left(\Sigma^{1L\,\prime}_{aa}(m^{l\,2}_a) + \Sigma^{1R\,\prime}_{aa}(m^{l\,2}_a)\right), \end{aligned} \tag{32}$$

$$\begin{aligned} \delta Z^{l_a}_R = {} & \Sigma^{\gamma R}_{aa}(m^{l\,2}_a) + m^{l\,2}_a \left(\Sigma^{\gamma L\,\prime}_{aa}(m^{l\,2}_a) + \Sigma^{\gamma R\,\prime}_{aa}(m^{l\,2}_a)\right) \\ & + m^l_a \left(\Sigma^{1L\,\prime}_{aa}(m^{l\,2}_a) + \Sigma^{1R\,\prime}_{aa}(m^{l\,2}_a)\right). \end{aligned} \tag{33}$$

Note that terms proportional to the masses are non-negligible only for the photonic transition [5]. The diagrams that enter the calculation are depicted on fig. 1. The left-right symmetry has for consequence the equality of the divergent parts of $\delta Z^{l_a}_L$ and $\delta Z^{l_a}_R$. We get the following simple result:

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[4] not to break renormalizability, we have to include all the diagrams. Nevertheless, we can simplify the finite parts of the radiative corrections, by assuming that all neutrinos are massless. The difference will be accounted for in numerics [10].

[5] since we consider only light particles, these terms are proportional to $\frac{(m^l_a)^2}{M^2}$, where M is a heavy boson mass. This can, obviously, be neglected.

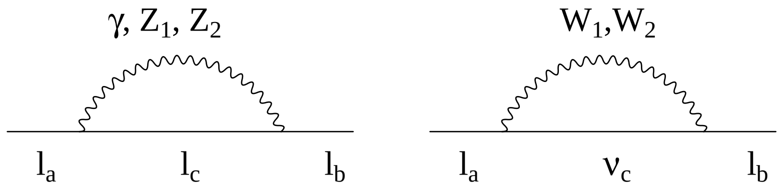

Fig. 1. Diagrams entering the charged lepton self-energy, with neglect of heavy neutrino contributions

$$
\begin{aligned}
\delta Z_L^{l_a} = \frac{\alpha}{4\pi}\Bigg(&-\Delta(m_a^l) - \frac{1}{2s_W^2}\Delta(M_{W_1}) - \frac{(A_L^{l1})^2}{4s_W^2c_W^2}\Delta(M_{Z_1}) - \frac{(A_L^{l2})^2}{4s_W^2c_W^2}\Delta(M_{Z_2}) \\
&-2\log\frac{\lambda^2}{m_a^{l\,2}} - 4 + \frac{(A_L^{l1})^2}{8s_W^2c_W^2} + \frac{(A_L^{l2})^2}{8s_W^2c_W^2} + \frac{1}{4s_W^2}\Bigg),
\end{aligned} \tag{34}
$$

$$
\begin{aligned}
\delta Z_R^{l_a} = \frac{\alpha}{4\pi}\Bigg(&-\Delta(m_a^l) - \frac{1}{2s_W^2}\Delta(M_{W_2}) - \frac{(A_R^{l1})^2}{4s_W^2c_W^2}\Delta(M_{Z_1}) - \frac{(A_R^{l2})^2}{4s_W^2c_W^2}\Delta(M_{Z_2}) \\
&-2\log\frac{\lambda^2}{m_a^{l\,2}} - 4 + \frac{(A_R^{l1})^2}{8s_W^2c_W^2} + \frac{(A_R^{l2})^2}{8s_W^2c_W^2} + \frac{1}{4s_W^2}\Bigg).
\end{aligned} \tag{35}
$$

The above do not depend on the lepton species, apart from the infrared logarithms, and even these will cancel in the photon vertex due to the electromagnetic Ward identity. The neutrino constants are quite similar. We do not give the diagrams, since they are analogous. The result is:

$$
\begin{aligned}
\delta Z_L^{\nu_a} = \delta Z_R^{\nu_a} = \frac{\alpha}{4\pi}\Bigg(&-\frac{1}{2s_W^2}\Delta(M_{W_1}) - \frac{(A_L^{\nu1})^2}{4s_W^2c_W^2}\Delta(M_{Z_1}) - \frac{(A_L^{\nu2})^2}{4s_W^2c_W^2}\Delta(M_{Z_2}) \\
&+\frac{(A_L^{\nu1})^2}{8s_W^2c_W^2} + \frac{(A_L^{\nu2})^2}{8s_W^2c_W^2} + \frac{1}{4s_W^2}\Bigg),
\end{aligned} \tag{36}
$$

Let us stress, that these constants are equal to one another due to the Majorana nature of the neutrinos.

If we were to have non-diagonal transitions, then renormalization of mixing matrices would be required as pointed out first by *Denner and Sack* [23], and later for the case of neutrinos by *Kniehl and Pilaftsis* [24]. Thanks to our simplifications, we do not have to bother about it right now.

Although in four-fermion processes we do not have to renormalize external photon lines, this is necessary to define the electric charge counter-term. Let us give the photon one-loop self-energy contribution [6] :

$$\begin{aligned}\Pi_{\gamma\gamma}(p^2) = \frac{\alpha}{4\pi}\Bigg(& -\frac{4}{3}\sum_{ferm.} Q_f^2\left(\Delta(m_f)p^2 + (2m_f^2+p^2)F(p,m_f,m_f) - \frac{p^2}{3}\right) \\ & +3\sum_{i=1,2}\left(\Delta(M_{W_i})p^2 + (p^2+\frac{4}{3}M_{W_i}^2)F(p,M_{W_i},M_{W_i})\right) \\ & -\frac{1}{3}\sum_{Higgs} Q_H^2\left(\Delta(M_H)p^2 + (p^2-4M_H^2)F(p,M_H,M_H) + \frac{2}{3}p^2\right)\Bigg) .\end{aligned} \tag{37}$$

One can check readily that this vanishes at zero momentum, which means that the photon remains massless. The derivative of this expression is the photon wave-function renormalization constant:

$$\begin{aligned}\delta Z^{\gamma} \equiv \Pi'_{\gamma\gamma}(0) = \frac{\alpha}{4\pi}\Bigg(& -\frac{4}{3}\sum_{ferm.} Q_f^2\left(\Delta(m_f)\right) + 3\sum_{i=1,2}\left(\Delta(M_{W_i}) + \frac{2}{9}\right) \\ & -\frac{1}{3}\sum_{Higgs} Q_H^2\left(\Delta(M_H)\right)\Bigg) .\end{aligned} \tag{38}$$

This should be compared to the SM result. We see that the fermions gave the same contribution, which was expected since the electromagnetic interaction is the same in all models. The term coming from the additional charged gauge boson is the same as the one for W_1, apart from the mass difference. The new addition is the term coming from charged Higgs loops. Its contribution is not numerically large due to logarithmic nature.

3.3 The Electric Charge Counter-Term

The electric charge is defined through the Thomson scattering amplitude. In practice it is measured in different processes, like the Hall or Josephson effect, but the former definition is closer to our perturbative methods. Thanks to the electromagnetic Ward identity, δe will be infrared divergence free.

[6] $i\Pi_{ba}$ is the transverse part of the irreducible contribution to the transition from gauge boson a to b, see [22] for the definition of the F function.

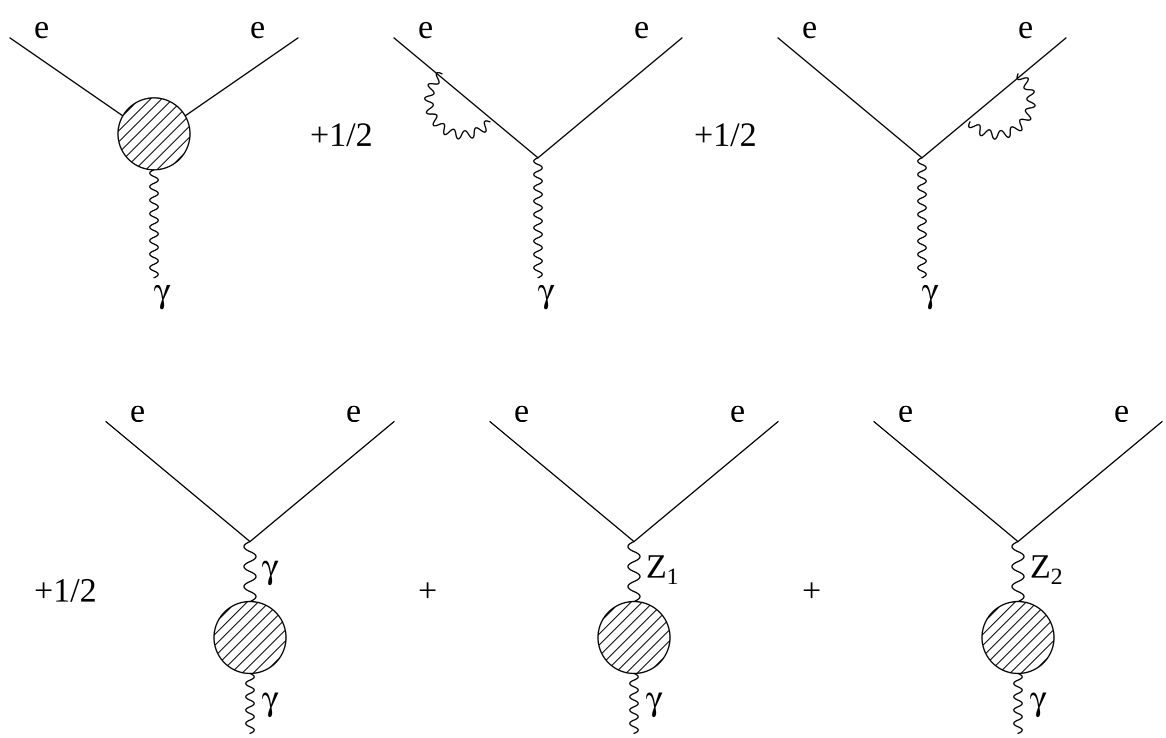

Fig. 2. Diagrams entering the electric charge counter-term.

The one-loop contribution to the γll amplitude, may only have a divergent vector part, otherwise it would not be multiplicatively renormalizable. It is thus an important check on the calculations to verify the vanishing of the divergence of the axial part. The diagrams entering this contribution are depicted on fig 2. Again, the left-right symmetry of the model causes the vanishing of the divergence of the axial part of the photon vertex corrections and the equality of those of the left and right lepton renormalization constants. Thus, what remains is a constraint on the γZ_1 and γZ_2 transitions at zero momentum:

$$\frac{1}{M_{Z_1}}\left(A_L^{l1}-A_R^{l1}\right)\Pi_{\gamma Z_1}(0)=\frac{1}{M_{Z_2}}\left(A_L^{l2}-A_R^{l2}\right)\Pi_{\gamma Z_2}(0). \tag{39}$$

Now, these transitions read:

$$\begin{aligned}\Pi_{\gamma Z_1}(0)=&\frac{\alpha}{4\pi}\frac{2}{s_W^2}\Big(\Delta(M_{W_1})(c/c_W+c_M s)M_{W_1}^2 \\ &+\Delta(M_{W_2})(\sqrt{2}s_g^n c_g^c-c_g^n s_g^c)M_{W_2}M_{Z_1}\Big)\,,\end{aligned} \tag{40}$$

$$\begin{aligned}\Pi_{\gamma Z_2}(0)=&\frac{\alpha}{4\pi}\frac{2}{s_W^2}\Big(\Delta(M_{W_1})(s/c_W-c_M c)M_{W_1}^2 \\ &+\Delta(M_{W_2})(\sqrt{2}c_g^n c_g^c+s_g^n s_g^c)M_{W_2}M_{Z_2}\Big)\,.\end{aligned} \tag{41}$$

The eq. 39 is satisfied, as we see. Since everything seems to be correct, we can

now calculate the electric charge counter-term. Let us remind that from fig. 2, follows that [7] :

$$\frac{\delta e}{e} = -\left(\Lambda_{\gamma ll}(0) + \frac{1}{2}(\delta Z_L^l + \delta Z_R^l) + \frac{1}{2}\delta Z^\gamma \right. \tag{42}$$
$$\left. + \frac{1}{4 s_W c_W}\left(\frac{1}{M_{Z_1}}\left(A_L^{l1} + A_R^{l1}\right)\Pi_{\gamma Z_1}(0) + \frac{1}{M_{Z_2}}\left(A_L^{l2} + A_R^{l2}\right)\Pi_{\gamma Z_2}(0)\right)\right)$$

Evaluating this expression leads to:

$$\frac{\delta e}{e} = \frac{\alpha}{4\pi}\left(\frac{2}{3}\sum_{ferm.} Q_f^2\left(\Delta(m_f)\right) + \sum_{i=1,2}\left(-\frac{7}{2}\Delta(M_{W_i}) - \frac{1}{3}\right) + \frac{1}{6}\sum_{Higgs} Q_H^2\left(\Delta(M_H)\right)\right). \tag{43}$$

The non-abelian couplings from both of the $SU(2)$ groups lead to a modification of the *QED* Ward identity:

$$\frac{\delta e}{e} = -\frac{1}{2}\delta Z^\gamma - \frac{\alpha}{4\pi}2\sum_{i=1,2}\Delta(M_{W_i}) \tag{44}$$

A similar result in the SM has been easily justified with use of an abelian Ward identity and a few other constraints. An analogous proof would be more difficult in this case.

Let us note at last, that due to a coincidence between the number of fermion generations and charge assignments, and the number of charged Higgses δe turns out to be finite. This was first noted by *Duka* [25] [8] .

3.4 The Weinberg Angle Counter-Term

The most difficult part of the renormalization scheme comes now. The approximations that $\kappa_2 = 0$, together with the remarks at the beginning of the present section, allow us to forget about the ζ angle. The ϕ angle counter-term

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[7] $-ie\Lambda_{\gamma ll}(0)$, is the one-loop correction to the γll vertex at zero momentum transfer, where l stands as usual for the lepton.

[8] This is an interesting observation, since it has consequences for the running of the electromagnetic coupling constant. Remark, that at some energy scale larger than the largest mass of the charged particles, α will cease running. This means that we will have problems embedding this left-right model in a GUT. Note that some parameters of the Higgs potential have been made vanishing by hand, and it has been argued that this may find its reason in some GUT model.

is needed only in neutral current processes. What remains, is the Weinberg angle. In principle, it can be expressed analytically through the gauge boson masses (but this leads to a fourth order equation). When $\kappa_2 = 0$, the appropriate expression is rather simple. It should not be used, however, to derive a counter-term, since the result would not take into account the counter-term to κ_2. The situation is fortunately simpler because δs_W^2 can be obtained from a system of linear equations.

Let us start by writing the following (see Eqs.(26,27)):

$$M_{W_2}^2 + M_{W_1}^2 = \frac{1}{2} g^2 (\kappa_+^2 + v_R^2), \tag{45}$$

$$M_{Z_2}^2 + M_{Z_1}^2 = \frac{1}{2} (g^2 \kappa_+^2 + 2 v_R^2 (g^2 + g'^2)), \tag{46}$$

$$M_{Z_2} M_{Z_1} = \frac{1}{2} g^2 \sqrt{1 + 2\frac{g'^2}{g^2}} \kappa_+ v_R. \tag{47}$$

To incorporate one-loop corrections, we have to expand these equations to first order:

$$\delta M_{W_2}^2 + \delta M_{W_1}^2 = \frac{1}{2} \delta g^2 (\kappa_+^2 + v_R^2) + \frac{1}{2} g^2 (\delta\kappa_+^2 + \delta v_R^2), \tag{48}$$

$$\delta M_{Z_2}^2 + \delta M_{Z_1}^2 = \frac{1}{2} \delta g^2 \kappa_+^2 + v_R^2 (\delta g^2 + \delta g'^2) + \frac{1}{2} g^2 \delta\kappa_+^2 + \delta v_R^2 (g^2 + g'^2), \tag{49}$$

$$\begin{aligned} M_{Z_2} \delta M_{Z_1} + M_{Z_1} \delta M_{Z_2} = & \frac{1}{2} \delta \left(g^2 \sqrt{1 + 2\frac{g'^2}{g^2}} \right) \kappa_+ v_R \\ & + \frac{1}{4} g^2 \sqrt{1 + 2\frac{g'^2}{g^2}} \left(\frac{v_R}{\kappa_+} \delta\kappa_+^2 + \frac{\kappa_+}{v_R} \delta v_R^2 \right), \end{aligned} \tag{50}$$

where:

$$\delta \left(g^2 \sqrt{1 + 2\frac{g'^2}{g^2}} \right) = \frac{1}{g^2 \sqrt{1 + 2g'^2/g^2}} \left((g^2 + g'^2) \delta g^2 + g^2 \delta g'^2 \right). \tag{51}$$

If $\delta\kappa_+^2 = \delta v_R^2 = 0$, then one could even take δs_W^2 from any of these equations. The fact is however, that we would have to include tadpole diagrams in gauge boson self-energies. This would be considerably more difficult, since it would have introduced the Higgs sector. We thus silently put the tadpoles to zero through appropriate renormalization constants, but use all of the equations to

eliminate $\delta\kappa_+^2$ and δv_R^2. Now, since the coupling constants are related to the Weinberg angle through:

$$g = \frac{e}{\sin\theta_W}, \quad g' = \frac{e}{\sqrt{\cos 2\theta_W}}, \tag{52}$$

the following result ensues:

$$\begin{aligned}
\delta s_W^2 = 2c_W^2 & \frac{(\delta M_{Z_2}^2 + \delta M_{Z_1}^2) - (\delta M_{W_2}^2 + \delta M_{W_1}^2)}{(M_{Z_2}^2 + M_{Z_1}^2) - (M_{W_2}^2 + M_{W_1}^2)} \\
& + \frac{1}{2}\frac{(M_{W_2}^2 + M_{W_1}^2)(\delta M_{Z_2}^2 + \delta M_{Z_1}^2) + (M_{Z_2}^2 + M_{Z_1}^2)(\delta M_{W_2}^2 + \delta M_{W_1}^2)}{\Big((M_{Z_2}^2 + M_{Z_1}^2) - (M_{W_2}^2 + M_{W_1}^2)\Big)^2} \\
& - \frac{1}{2}\frac{(2M_{Z_1}^2 + M_{Z_2}^2)\delta M_{Z_1}^2 + (2M_{Z_2}^2 + M_{Z_1}^2)\delta M_{Z_2}^2}{\Big((M_{Z_2}^2 + M_{Z_1}^2) - (M_{W_2}^2 + M_{W_1}^2)\Big)^2}.
\end{aligned} \tag{53}$$

Although it looks symmetric, this is also much more complicated than in the SM. The last important point is this equation:

$$\Big((M_{Z_2}^2 + M_{Z_1}^2) - (M_{W_2}^2 + M_{W_1}^2)\Big) = \frac{g^2}{2c_M^2 c_W^2} v_R^2. \tag{54}$$

It sets the scale of the radiative corrections, which will be the subject of study of the next section.

3.5 Methods and Cross-Checks

We have presented the necessary parts of the renormalization scheme. When applying the formulae, and most of all the δs_W^2 constant we are facing the problem of dealing with enormous numbers of diagrams. To make this study feasible, certain computer methods have been developed. We would like to describe them in some detail.

Most of the calculations have been performed with the program FORM [26]. This includes the basic one-loop diagrams with undefined coupling constants. They have been tested on the Standard Model. All of the results given in standard works on radiative corrections to the SM have been recovered. The vertex and fermion self-energy corrections have been coded by analogy to the SM, and it has been verified that they lead to the same results in appropriate limits. It has been thus assumed, that the formulae are correct, and that the only source of problems would be the couplings, statistical factors and signs in the gauge boson self-energies. Because of the large Higgs sector, a special

program has been written in C, to parse lagrangians and produce input for FORM in terms of the previously tested functions. It again, has been able to recover SM results. To verify the correctness of the results for the full LR model, we used a set of constraints on the divergent parts of the self-energies following from the relation between its broken and its unbroken phases [9] [10]:

$$\left(\delta Z^{W_2}\right)_{div.} = \left(\delta Z^{W_1}\right)_{div.}, \tag{56}$$

$$v_1^2\left(\delta Z^{Z_2} - (x_2^2+y_2^2)\delta Z^{W_2}\right)_{div.} = v_2^2\left(\delta Z^{Z_1} - (x_1^2+y_1^2)\delta Z^{W_1}\right)_{div.}, \tag{57}$$

and

$$\left(\delta Z^{Z_1}\right)_{div.} = (x_1^2+y_1^2)\left(\delta Z^{W_1}\right)_{div.} + \frac{v_1^2}{(c_M^2 c_W^2)}\left(\delta Z^{\gamma} - 2s_W^2\delta Z^{W_1}\right)_{div.}, \tag{58}$$

$$\left(\delta Z^{Z_2}\right)_{div.} = (x_2^2+y_2^2)\left(\delta Z^{W_2}\right)_{div.} + \frac{v_2^2}{(c_M^2 c_W^2)}\left(\delta Z^{\gamma} - 2s_W^2\delta Z^{W_2}\right)_{div.}. \tag{59}$$

The last and certainly the hardest was to verify the finiteness of the renormalized W_1 vertex, which would prove the finiteness of the muon decay amplitude. This has also been successfully done.

4 Muon Decay at One-Loop

We have to decompose all of the corrections into classic electromagnetic and the weak ones. Let us remind, that the muon life time is described through the Fermi coupling constant G_F, which, for historical reasons, does not contain electromagnetic radiative contributions to the Fermi theory.

The method of calculation has been devised by *Sirlin* [21]. He noted, that if

[9] the divergent parts of the gauge boson wave function renormalization constants can be found without any explicit assumptions on boson renormalization equations. They are simply the parts of the self-energies proportional to the momentum squared.

[10] To simplify notation we identify the neutral sector mixing matrix given in Eq.(22) with the following matrix

$$\begin{pmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ v_1 & v_2 & v_3 \end{pmatrix}. \tag{55}$$

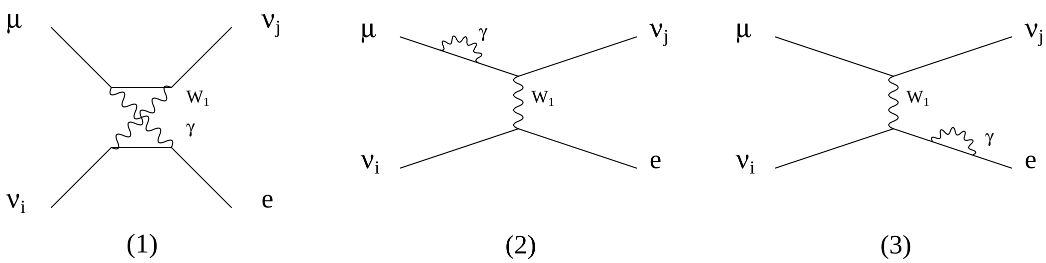

Fig. 3. Diagrams entering the electromagnetic corrections to the Fermi theory.

we write the photon propagators in the muon and electron wave function, in the following way:

$$\frac{1}{k^2} = \frac{1}{k^2 - M_{W_1}^2} - \frac{M_{W_1}^2}{k^2 - M_{W_1}^2}\frac{1}{k^2}, \tag{60}$$

then diagrams of fig 3 with the second term of the above equation, are, to an excellent approximation (of the order $\frac{\alpha}{4\pi}\frac{m_\mu^2}{M_{W_1}^2}$) equal to the electromagnetic corrections to the Fermi theory without bremsstrahlung. One should, therefore, calculate radiative corrections to G_F by simply neglecting these diagrams, but using a massive photon on the external lepton lines [11] . This method has an additional advantage, that we will not have to regularize the photonic corrections, since everything will be infrared finite.

Up to this point everything was the same as in the SM. One more point that is similar, is that we can introduce a quantity, namely Δr, that will contain all the radiative corrections:

$$G_F = \frac{\pi\alpha}{\sqrt{2}}\frac{1}{M_{W_1}^2 s_W^2}\left(1 + \Delta r\right). \tag{61}$$

We can decompose Δr into the following terms:

$$\Delta r = \frac{Re\Pi_{W_1W_1}(M_{W_1}^2) - \Pi_{W_1W_1}(0)}{M_{W_1}^2} - \Pi'_{\gamma\gamma}(0) - \frac{\delta s_W^2}{s_W^2} + \delta_{V+B}, \tag{62}$$

$\Pi'_{\gamma\gamma}$ comes from δe, and the rest of contributions is included in δ_{V+B}, which stands for vertex and box corrections. The first three are oblique corrections.

One might wonder, why we did not perform a Dyson resummation of the W_1 propagator. This is due to the requirement of gauge invariance. Written in the above form the amplitude satisfies it, but if we treat the bosonic propagator in a specific way, this gauge invariance will be lost. In SM model analyses, it is

[11] note however that the photonic corrections to vertices and boson wave functions should contain a massless photon.

usual to resum fermion loop contributions, since they can be shown to be gauge invariant. Here the situation is more interesting. Apart from large fermion loops, also Higgs loops can be important. Therefore resumming fermion loops will lead to an asymmetry in the treatment of dominating effects. There is a way to solve the problem. We will not do it here, but to get better results one should use methods of the type of pinch technique [27] or background field [28]. Further studies of these, applied to the LR model, will certainly be necessary[9].

4.1 Top quark, Higgses and Oblique Corrections

The leading heavy particle contributions concentrate in δs_W^2. Let us note that utmost importance is attached to the top quark and Higgs in the SM.

There is a large difference between the two models. In our case, we have two scales, one of them corresponds to the SM breaking scale, but the second must be much larger. It so happens, that δs_W^2 corresponds to the second one. Let us remind, that it contains only terms proportional to the ratio of mass renormalization constants and v_R^2 (see Eqs.(53,54)).

In the Standard Model, the top quark came into play through the $\Delta\rho$ part of δs_W^2. Due to the existence of one scale, its contribution is, as we know [2]:

$$(\Delta r)_{SM}^{top} = -\frac{c_W^2}{s_W^2}(\Delta\rho)^{top}, \tag{63}$$

where:

$$(\Delta\rho)^{top} = \frac{\sqrt{2}G_F}{16\pi^2}3m_t^2. \tag{64}$$

In the LR model, the situation is much different, namely:

$$(\Delta r)_{LR}^{top} = \frac{\sqrt{2}G_F}{8\pi^2}c_W^2\left(\frac{c_W^2}{s_W^2}-1\right)\frac{M_{W_1}^2}{M_{W_2}^2-M_{W_1}^2}3m_t^2. \tag{65}$$

For a W_2 boson mass of the order of 400 GeV or larger, this contribution is smaller than the SM logarithmic terms. This simply means, that with realistic masses of additional gauge bosons, we loose the quadratic mass dependence of the oblique corrections on the top mass. Let us note, that one of the beautiful evidences for the correctness of the SM, came from the excellent agreement between the predicted top mass (from oblique corrections) and the actually

measured at Fermilab. We have to acknowledge, that the LR model cannot display this quality [12] .

Let us consider the Higgses. In the SM there is custodial symmetry, which induces a cancellation of the quadratic mass term of the Higgs. The remaining logarithms are weak enough not to allow for a precise prediction of the still unknown Higgs mass. Remark that quadratic mass terms put stringent bounds on the allowed masses. It could be feared, that due to the lack of custodial symmetry, large Higgs masses, as required by $FCNC$ bounds, would not be allowed by the oblique corrections. The LR model in this minimal version would be ruled out. Fortunately for us, these terms occur in ratios with the large symmetry breaking scale. Therefore, the risk is lessened. We can give a typical Higgs contribution. This one is for the lightest, supposedly analogue of the SM Higgs:

$$(\Delta r)_{LR}^{lightest\ Higgs} = \frac{\sqrt{2}G_F}{48\pi^2}\left(\frac{M_{W_1}^2}{M_{W_2}^2}\frac{c_W^2}{s_W^2}(1-2s_W^2) + \frac{M_{W_1}^2}{M_{Z_2}^2}\frac{1}{s_W^2}(4c_W^2-1)\right) M_{H_0^0}^2. \tag{66}$$

The remaining Higgses give terms of comparable structure.

4.2 Heavy Neutrinos

Contrary to the top, heavy neutrinos give important contributions proportional to squares of the masses. We have:

$$(\Delta r)_{LR}^{N} = \sum_{N=heavy} \frac{\sqrt{2}G_F}{16\pi^2} c_M^2 c_W^2 \frac{M_{W_1}^2}{M_{W_2}^2 - M_{W_1}^2} m_N^2. \tag{67}$$

The exact contribution depends heavily on the relation between neutrino and W_2 masses. If we assume (three) heavy neutrinos, to be of the order of M_{W_2}, then from Eqs.(63,67) we have ($m_N \simeq M_{W_2} >> M_{W_1}$)

$$|(\Delta r)_{LR}^{N}/(\Delta r)_{SM}^{top}| \simeq c_M^2 s_W^2 \frac{M_{W_1}^2}{m_t^2} \simeq 0.02, \tag{68}$$

[12] This is a feature of all models with $\rho \neq 1$ as first noted by *Jegerlehner* [29].

so the heavy neutrino contribution of oblique corrections to Δr is smaller than the SM top one. However, already with $m_N \simeq 7M_{W_2}$ this is comparable [13]. Interestingly, neutrino contributions come with the opposite sign to the SM top quark one, similarly as in [31].

Finally let us comment shortly on heavy neutrino contributions to vertices and boxes. In the SM, vertex corrections were not negligible, but small and under good control. The reason was, that as mass terms of the leptons were unimportant, they could be parametrized as functions of the gauge boson masses. Moreover, Higgs diagrams could be neglected altogether. The situation changes drastically in the LR model. The corrections now depend seriously not only on masses, but also on the mixing matrices. The largest terms, generated by Higgs diagrams, can in principle exhibit a quadratic dependence on the neutrino masses. This can be easily seen, with some dimensional analysis. Fortunately, it has been proved in the case of the SM with additional right handed singlets [3], that these terms cancel with similar contributions from the external wave functions. Since the proof can be made by explicit calculation [14], and in this respect nothing changes if we go over to the LR model, here also these terms cancel. Remain, however, contributions proportional to logarithms of ratios of neutrino masses to light gauge boson masses. They are not negligible, but to make any predictions, concrete mixing matrices and masses must be considered [10].

The situation is more complex with boxes. This time, there is nothing that contributions proportional to squares of neutrino masses could cancel against. It can be conjectured, that these terms will put stringent bounds on allowed neutrino masses and mixings.

5 Conclusions and Outlook

We have performed a full one loop calculation of a physical process, the muon decay, in the framework of the left-right symmetric model. The calculation involved a huge number of diagrams. It has thus been performed using the computer program FORM, and custom programs.

A simple renormalization scheme has been developed for four-fermion processes. It has been shown to work, by a direct check of divergence cancellation. Its relation to the Standard Model results has been studied.

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[13] Let us note however, that such heavy neutrinos are at the edge of perturbation theory where (assuming neutrino Yukawa couplings to be smaller than one), a natural limit can be derived, $m_N \leq 2M_{W_2}/g$ [30]

[14] and does not depend on anything more than just properties of one-loop scalar functions.

Later, contributions of heavy particles through oblique corrections have been discussed. It has been shown that the strong dependence on the top quark mass, so famous in the SM, has been lost. The size of Higgs and heavy neutrino diagrams has been demonstrated not to exceed reasonable bounds, thereof leading to the conclusion that the model is not trivially ruled out.

At last, we have to note, that Δr of the LR model is much different, apart from the $\Delta\alpha$ contributions, from its SM counterpart. It means, that if one makes bounds from tree level diagrams, corrected by SM Δr, the result is only a rough approximation of reality. In fact as seen in the last section, in principle Δr can be anything.

As an outlook of the future, we have to state that many issues remain to be considered. First and foremost, fits to low- and high-energy processes are now possible [10]. A lot of effort is nevertheless required on the phenomenological side. Without wise guesses on the possible heavy neutrino sector, it would be impossible to derive any numerics. Thus, it again shows, that light states are nontrivially entangled with unobservable heavy ones.

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